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An Analysis of the Production Leader in the Stackelberg Model

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Abstract: The paper analyzes the situation of a duopoly in which each competitor does not have complete information about the role assumed by the other.

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1. Introduction

Oligopole represents the situation of a market where there is a small number of bidders (at least two) of a non-substitutable good and a sufficiently large number of consumers. Oligopole composed of two producers is called duopoly.

Considering two competing companies A and B that produce the same normal good, we propose to analyze each of them in response to the activity of the other firm.

Each of them when determining their production level and selling price will consider the production and price of the other. If one of the two companies first establishes the price or the quantity produced, the other one adjusting for it, then it will be called the price leader, respectively the production leader, the second firm called the price satellite or the production satellite.

2. The Analysis

Companies A and B have incomplete information about each other. They will consider the other firm as a production leader or satellite.

Consider for the beginning that the company A is considered to be leader of quantity. If it produces Q_A good units, then B, considered by A as a satellite, will adjust its output after A producing $Q_B=f(Q_A)$ units of good. The sales price is dependent on the total quantity of products placed on the market. So let's say: $p=p(Q_A+Q_B)$ the price per unit of product.

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Company A must set a production level based on B's reaction, as this will result in the product's sales price through its output. Similarly, from the point of view of A, company B will adjust its production level according to its own, because at a higher or lower level the price will change and therefore the profit of the respective company.

We assume that the two marginal costs of A and B are constant (in the short term, marginal cost variations being very small, the assumption is not absurd): $Cm_A=\alpha$ and $Cm_B=\beta$. Also, the price function (the inverse function of the demand) will be of the form: $p(Q)=a-bQ$, $a,b>0$. So let the profit of the production leader A:

$$\Pi_A(Q_A)=p\left(Q_A+Q_B\right)Q_A-CT_A(Q_A)=-bQ_A^2-bQ_AQ_B+aQ_A-CT_A(Q_A)$$

Because $Q_B=f(Q_A)$ we have:

$$\Pi_A(Q_A)=p\left(Q_A+f(Q_A)\right)Q_A-CT_A(Q_A)=-bQ_A^2-bQ_Af(Q_A)+aQ_A-CT_A(Q_A)$$

Consider the satellite's profit as well:

$$\Pi_B(Q_B)=p\left(Q_A+Q_B\right)Q_B-CT_B(Q_B)=-bQ_B^2-bQ_AQ_B+aQ_B-CT_B(Q_B)$$

The extreme condition for A's profits is:

$$\frac{\partial\Pi_A(Q_A)}{\partial Q_A}=-2bQ_A-bf(Q_A)-bQ_Af'(Q_A)+a-\alpha=0$$

and for B:

$$\frac{\partial\Pi_B(Q_B)}{\partial Q_B}=-2bQ_B-bQ_A+a-\beta=0$$

Considering therefore the production of the Q_A leader as being given, it results that the satellite satisfies the condition:

$$Q_B=\frac{a-\beta-bQ_A}{2b}$$

By replacing Q_B's above expression in the condition of maximizing A's profit, it results for $f(Q_A)=\frac{a-\beta-bQ_A}{2b}$: $-2bQ_A-b\frac{a-\beta-bQ_A}{2b}+\frac{bQ_A}{2}+a-\alpha=0$ from where:

$$Q_A^*=\frac{a-2\alpha+\beta}{2b}$$

and now: $Q_B^*=\frac{a-\beta-bQ_A^*}{2b}=\frac{a+2\alpha-3\beta}{4b}$.

We have therefore: $Q_A^*-Q_B^*=\frac{a-6\alpha+5\beta}{4b}$.

The assumption that A is the leader of quantity will be real if $Q_A^*-Q_B^*\geq 0$ that is: $a\geq 6\alpha-5\beta$.

In the case $\alpha>\beta$, if A will be leader of quantity, we have $a\geq\beta$.

If B, however, assumes the role of production leader, considering A as satellite, from the above calculations, in which we will permute all the A and B-related indicators, we will get:

$$Q_B^{**} = \frac{a - 2\beta + \alpha}{2b},\ Q_A^{**} = \frac{a + 2\beta - 3\alpha}{4b}, \text{ from where: } Q_B^{**} - Q_A^{**} = \frac{a - 6\beta + 5\alpha}{4b}.$$

Therefore assuming that B is the leader of quantity will be real if: $Q_B^{**} - Q_A^{**} \geq 0$ that is: $a \geq 6\beta - 5\alpha$.

In the case $\alpha<\beta$ then if B is the leading quantity we have $a\geq\alpha$.

In order to determine the profits of the two firms, from the fact that marginal costs $CmA=\alpha$ and $CmB=\beta$ results after a simple integration: $CT_A(Q)=\alpha Q+\gamma$, $CT_B(Q)=\beta Q+\delta$, $\alpha,\beta,\gamma,\delta\geq0$.

As companies do not have complete information about each other, we will investigate the four situations that may arise:

Case 1: A - leader, B - leader

In this case, A takes the leading role and B is also a leader.

We have in this case: $Q^*_{A/A-leader,B-leader} = \frac{a - 2\alpha + \beta}{2b}$, $Q^*_{B/A-leader,B-leader} = \frac{a + \alpha - 2\beta}{2b}$ from where:

$$p_{A-leader,B-leader} = p\left(Q^*_{A/A-leader,B-leader} + Q^*_{B/A-leader,B-leader}\right) = \frac{\alpha + \beta}{2}, \text{ and:}$$

$$\Pi_{A/A-leader,B-leader}\left(Q^*_{A/A-leader,B-leader}\right) = \frac{(\alpha - \beta)(-a + 2\alpha - \beta)}{4b} - \gamma$$

$$\Pi_{B/A-leader,B-leader}\left(Q^*_{B/A-leader,B-leader}\right) = \frac{(\alpha - \beta)(a + \alpha - 2\beta)}{4b} - \delta$$

Case 2: A - leader, B - satellite

In this case, A takes the leading role and B is considered a satellite of A

We have in this case: $Q^*_{A/A-leader,B-satellite} = \frac{a - 2\alpha + \beta}{2b}$, $Q^*_{B/A-leader,B-satellite} = \frac{a + 2\alpha - 3\beta}{4b}$ from where:

$$p_{A-leader,B-satellite} = p\left(Q^*_{A/A-leader,B-satellite} + Q^*_{B/A-leader,B-satellite}\right) = \frac{a + 2\alpha + \beta}{4}, \text{ and:}$$

$$\Pi_{A/A-leader,B-satellite}\left(Q^*_{A/A-leader,B-satellite}\right) = \frac{(a - 2\alpha + \beta)^2}{8b} - \gamma$$

$$\Pi_{B/A-leader,B-satellite}\left(Q^*_{B/A-leader,B-satellite}\right) = \frac{(a + 2\alpha - 3\beta)^2}{16b} - \delta$$

Case 3: A - satellite, B - leader

In this case, B assumes the leading role and A is considered a satellite of B.

We have in this case: $Q^*_{A/A-satellite,B-leader} = \frac{a - 3\alpha + 2\beta}{4b}$, $Q^*_{B/A-satellite,B-leader} = \frac{a + \alpha - 2\beta}{2b}$ from where:

$$p_{A-satellite,B-leader} = p\left(Q^*_{A/A-satellite,B-leader} + Q^*_{B/A-satellite,B-leader}\right) = \frac{a+\alpha+2\beta}{4}$$, and:

$$\Pi_{A/A-satellite,B-leader}\left(Q^*_{A/A-satellite,B-leader}\right) = \frac{\left(a-3\alpha+2\beta\right)^2}{16b} - \gamma$$

$$\Pi_{B/A-satellite,B-leader}\left(Q^*_{B/A-satellite,B-leader}\right) = \frac{\left(a+\alpha-2\beta\right)^2}{8b} - \delta$$

Case 4: A - satellite, B - satellite

In this case, both A and B assume the satellite role of the other.

We have in this case: $Q^*_{A/A-satellite,B-satellite} = \frac{a-3\alpha+2\beta}{4b}$, $Q^*_{B/A-satellite,B-satellite} = \frac{a+2\alpha-3\beta}{4b}$ from where:

$$p_{A-satellite,B-satellite} = p\left(Q^*_{A/A-satellite,B-satellite} + Q^*_{B/A-satellite,B-satellite}\right) = \frac{2a+\alpha+\beta}{4}$$, and:

$$\Pi_{A/A-satellite,B-satellite}\left(Q^*_{A/A-satellite,B-satellite}\right) = \frac{\left(2a-3\alpha+\beta\right)\left(a-3\alpha+2\beta\right)}{16b} - \gamma$$

$$\Pi_{B/A-satellite,B-satellite}\left(Q^*_{B/A-satellite,B-satellite}\right) = \frac{\left(2a+\alpha-3\beta\right)\left(a+2\alpha-3\beta\right)}{16b} - \delta$$

We will then determine the short-term profit situation in the above cases.

Consider the situation of A. What kind of role does it take to assume A?

If B would be the leader then, if A also assumed the leading role:

$$\Pi_{A/A-leader,B-leader} = \frac{\left(\alpha-\beta\right)\left(-a+2\alpha-\beta\right)}{4b} - \gamma$$

If A would be considered a satellite of B then:

$$\Pi_{A/A-satellite,B-leader} = \frac{\left(a-3\alpha+2\beta\right)^2}{16b} - \gamma$$

The difference between the two profits is:

$$\Delta = \Pi_{A/A-leader,B-leader} - \Pi_{A/A-satellite,B-leader} = -\frac{\left(a-\alpha\right)^2}{16b} < 0.$$

Therefore, if B takes the leading role, A will have to take on the role of satellite to maximize its profit.

If now B would consider satellite then, if A assumes the leading role:

$$\Pi_{A/A-leader,B-satellite} = \frac{\left(a-2\alpha+\beta\right)^2}{8b} - \gamma$$

But if A assumes a satellite role, not knowing that B has also assumed this role:

$$\Pi_{A/A-satellite,B-satellite} = \frac{\left(2a-3\alpha+\beta\right)\left(a-3\alpha+2\beta\right)}{16b} - \gamma$$

The difference between the two profits is:

$$\Delta = \Pi_{A/A-leader,B-satellite} - \Pi_{A/A-satellite,B-satellite} = \frac{(a-\alpha)(\alpha-\beta)}{16b}.$$

How $(a-\alpha)(\alpha-\beta)>0 \Leftrightarrow \alpha \in (\min(a,\beta),\max(a,\beta))$ the following results:

- If B is considered satellite, if the marginal cost of A: $\alpha \in (\min(a,\beta),\max(a,\beta))$ then A must take the leading role;
- If B is considered satellite, if the marginal cost of A: $\alpha \notin (\min(a,\beta),\max(a,\beta))$ then A must assume a satellite role.

Consider the situation of B. Now what role do we have to assume B?

If A would consider himself a leader then, if B would also assume the leading role:

$$\Pi_{B/A-leader,B-leader} = \frac{(\alpha-\beta)(a+\alpha-2\beta)}{4b} - \delta$$

If B would be considered a satellite of A then:

$$\Pi_{B/A-leader,B-satellite} = \frac{(a+2\alpha-3\beta)^2}{16b} - \delta$$

The difference between the two profits is:

$$\Delta = \Pi_{B/A-leader,B-leader} - \Pi_{B/A-leader,B-satellite} = -\frac{(a-\beta)^2}{16b} < 0.$$

Therefore, if A takes the leading role, B will have to take on the role of satellite to maximize its profit.

If now it would consider satellite then, if B assumed the leading role:

$$\Pi_{B/A-satellite,B-leader} = \frac{(a+\alpha-2\beta)^2}{8b} - \delta$$

If B, however, assumes a satellite role, not knowing that A has also assumed this role:

$$\Pi_{B/A-satellite,B-satellite} = \frac{(2a+\alpha-3\beta)(a+2\alpha-3\beta)}{16b} - \delta$$

The difference between the two profits is:

$$\Delta = \Pi_{B/A-satellite,B-leader} - \Pi_{B/A-satellite,B-satellite} = \frac{(a-\beta)(-\alpha+\beta)}{16b}$$

How $(a-\beta)(-\alpha+\beta)>0 \Leftrightarrow \beta \in (\min(a,\alpha),\max(a,\alpha))$ the following results:

- If A is considered satellite, if the marginal cost of B: $\beta \in (\min(a,\alpha),\max(a,\alpha))$ then B must take the leading role;
- If A is considered satellite, if the marginal cost of B: $\beta \notin (\min(a,\alpha),\max(a,\alpha))$ then B must assume a satellite role.

3. Conclusions

- If the marginal cost of A is lower than the marginal cost of B ($\alpha<\beta$) then:

  - if $\alpha<a$ then if B is considered to be a satellite, A will also have to be considered satellite; if $\beta<a$ then if A is considered a satellite, B will have to be considered a leader; if $\beta>a$ then if A is considered to be satellite, B will also have to be considered satellite;

  - if $\alpha>a$ then if B is considered as a satellite, A will have to be considered a leader; if $\beta<a$ then if A is considered to be a satellite, B will have to be considered leader, but what can not be because of the fact that in this case: $a\geq\alpha$; if $\beta>a$ then if A is considered to be satellite, B will also have to be considered satellite;

  - if B takes the lead role, A will have to take on the role of satellite. By analogy, if A takes the lead role, B will have to assume the role of satellite.

- If the marginal cost of A is greater than the marginal cost of B ($\alpha>\beta$) then:

  - if $\alpha>a$ then if B is considered as a satellite, A should also be considered satellite; if $\beta>a$, then if A is considered a satellite, B will have to be considered a leader; if $\beta<a$ then if A is considered to be satellite, B should also be considered satellite;

  - if $\alpha<a$ then if B is considered a satellite, A will have to be considered a leader; if $\beta<a$, then if A is considered a satellite, B will have to be considered a leader; if $\beta<a$ then if A is considered to be satellite, B should also be considered satellite;

  - if B takes the lead role, A will have to take on the role of satellite. By analogy, if A takes the lead role, B will have to assume the role of satellite.

- If the marginal cost of A is equal to the marginal cost of B ($\alpha=\beta$) then if one of them is considered satellite, the other is indifferent to whether it is a leader or a satellite.

4. References

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