

THEORETICAL GROUND OVER THE COBWEB MODEL

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Abstract: *We propose a theoretical landing of the linear Cobweb model and effectuation of some numerous simulations which help of a onformatic product with multiple applications in economical research, which is Maple. We are ready to supply graphic programs to those who are interested.*

Keywords: *cobweb model, numerous simulations, maple program*

Presentation of Cobweb model

On some markets, also farming ones but also the ones of industrial goods with a long cycle of fabrication, the offer can't extend immediately to meet a greater growth. This way, a harvest must be planted first, then growth and recollected after, actions that ask for a certain period of time.

Cobweb model is the one who takes in consideration the same time delay of the offer reaction at the modifications of demand from a certain market, through the presumption that the size of the quantity offered now Q_t^{of} depending on the price from a previous period P_{t-1} in other order of meaning $Q_t^{of} = f(P_{t-1})$ where the basics show a period of time. The consumer demand on the same product market Q_t^{cer} depends on the current price, $Q_t^{cer} = f(P_t)$.

In case of a linear model of the market forces we will have:
 $Q_t^{cer} = a + bP_t$ and $Q_t^{of} = c + dP_{t-1}$

Where a, b, c are the specific function parameters of the demand and supply, and the normal goods b is possibly negative.

The balance of the market involves equalisation of demand and supply, which says:

$$Q_t^{cer} = Q_t^{of} \Rightarrow a + bP_t = c + dP_{t-1} \Rightarrow P_t = \left(\frac{c-a}{b}\right) + \frac{d}{b} P_{t-1}$$

The last relation shows a difference equation first order, because the prices are different with only one time unit.

In legal terms this equation can be generalized like: $X_t = \alpha + \beta X_{t-1}$, where x shows the variable what modifies in certain time, and α & β are constant measures like: $\alpha = (c-a)/b$ si $\beta = d/b$.

The solution of a different equation first order has two components:

1) The balance solution: in Cobweb model it is like the balance of price for a long period of time. As the balance price is the same in every period of time, it means that $P_t = P_{t-1}$, what means that the balance solution represents a constant measure in connection with variable adjustment which modifies in time.

We designate P^* balance price for long period which maintains in every period so:
 $P^* = P_t = P_{t-1}$, and substitute in difference equation $P_t = \left(\frac{c-a}{b}\right) + \frac{d}{b} P_{t-1}$ we will have: $P^* = \left(\frac{c-a}{b}\right) + \frac{d}{b} P^*$
 $P^*, P^* = \frac{a-c}{d-b}$, in equal mode and with the balance price with only one period.

2) The complementary solution:-name the way which the variable, the price of Cobweb model modifies from the balance solution by time. The difference equation $P_t = \left(\frac{c-a}{b}\right) + \frac{d}{b}P_{t-1}$, can be written like $P_t = \frac{d}{b}P_{t-1}$, because the first element is not changing in time. We presume that $P_t = Ak^t$ where A and k are constants; this function applying for all t values, so $P_{t-1} = Ak^{t-1}$, and substituting the prices in difference equation shorten we obtain: $Ak^t = \frac{d}{b}Ak^{t-1}$. The value of A can be shown by knowing a certain measure of the price from a certain period of time.

This way, the final solution of a difference equation Cobweb model will be:

$$P_t = \text{balance solution} + \text{complementary solution: } P_t = \left(\frac{a-c}{d-b}\right) + A\left(\frac{d}{b}\right)^t$$

3) Numerical simulations

The final form of the model depends on the value of report d/b, which for values different then 0 of A will create three situations:

1) If $\left|\frac{d}{b}\right| < 1$, then $\left(\frac{d}{b}\right)^t \rightarrow 0$ as so $t \rightarrow \infty$. This situation is registered on a stabil market, as so the deviation from the balance price is becoming smaller. We impose the absolute size of the report because b is negativ. See figure 1 and 2.

Fig 1 Modelul Cobweb de tip convergent $k < 1$

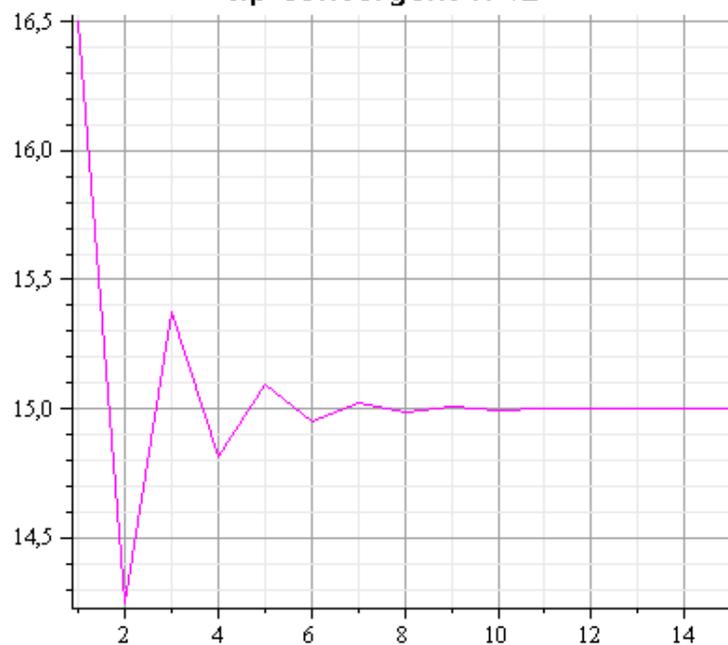
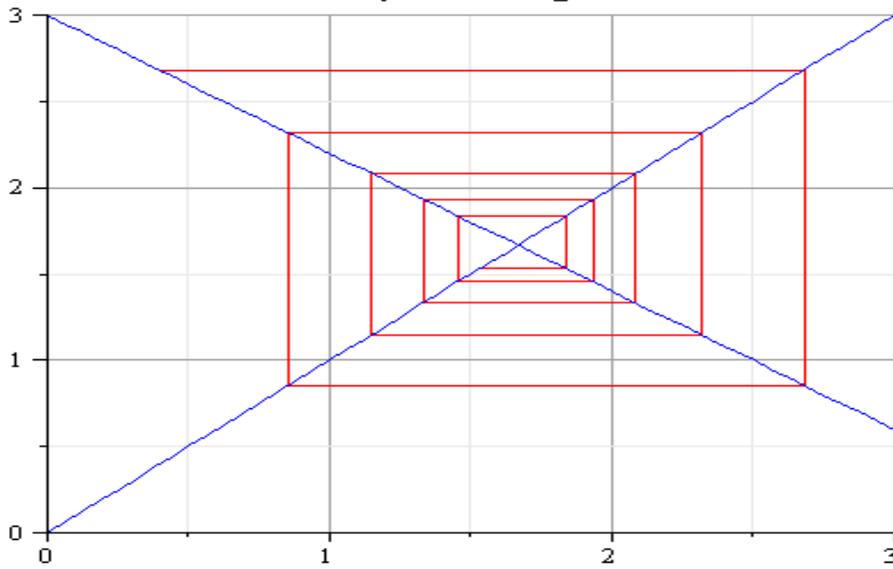


Figura 2 Modelul Cobweb de tip convergent



2) If $\left|\frac{d}{b}\right| > 1$, then $\left(\frac{d}{b}\right)^t \rightarrow \infty$ as so $t \rightarrow \infty$. This situation is registered on a unstable market. During the time the price will deviate from his balance value, with a bigger size, after a initial deviation. See figure 3 and 4.

Fig 3 Modelul Cobweb de tip divergent $k > 1$

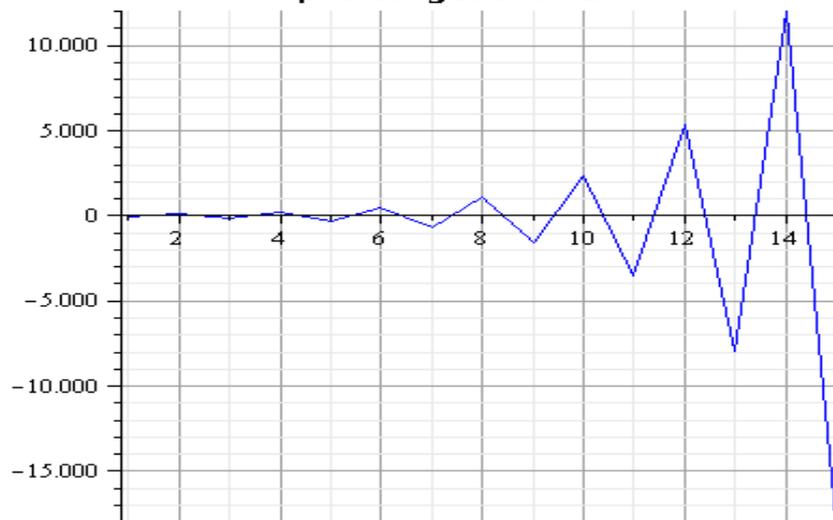
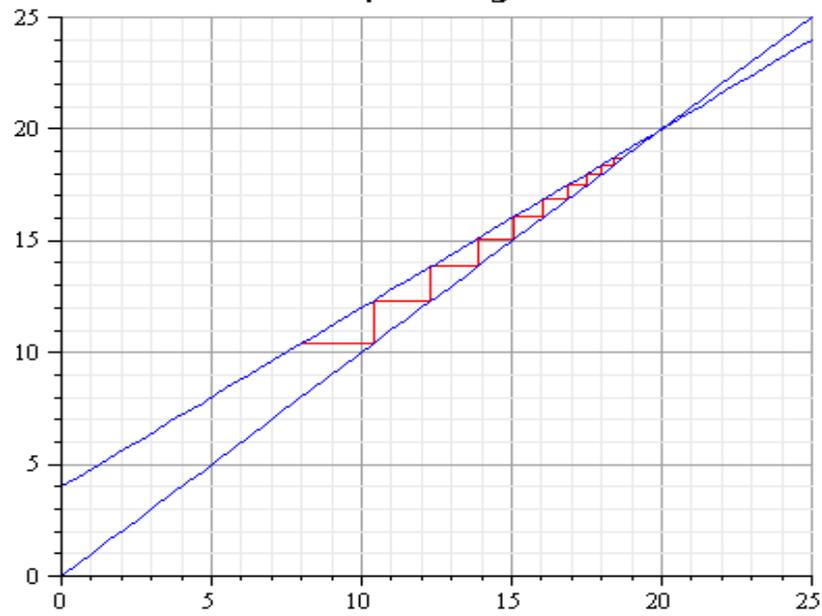


Figura 4 Modelul cobweb de tip divergent



3) If $\left|\frac{d}{b}\right|=1$, then $\left|\left(\frac{d}{b}\right)^t\right|=1$ as so $t \rightarrow \infty$. This situation will be registered on a fluctuant market, the price will change between two levels. See figure 5 and 6.

Fig 5 Modelul Cobweb de tip fluctuant k=1

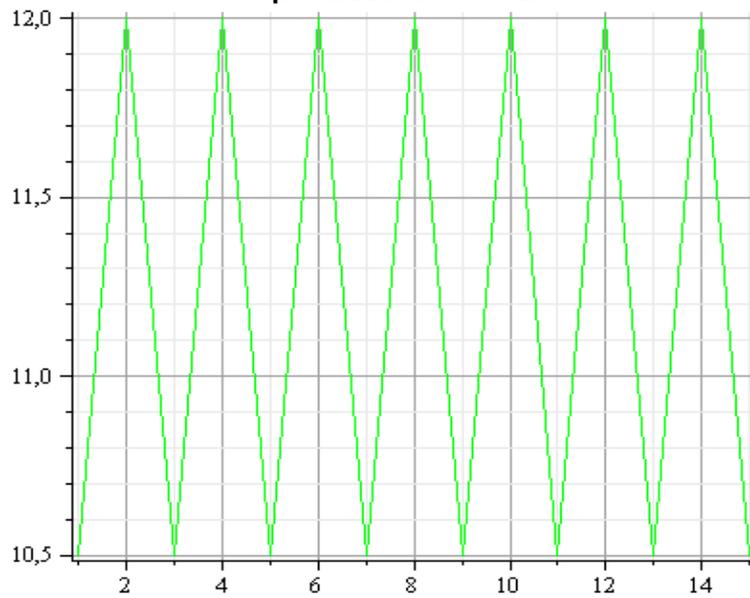
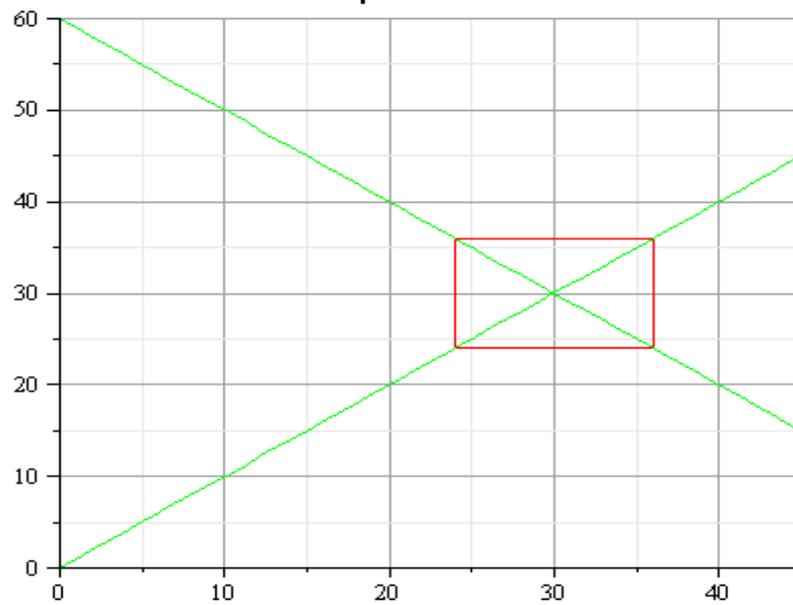


Figura 6 Modelul cobweb de tip fluctuant



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