Fundamental Relations in Monopolistic Competition of Dixit-Stiglitz Type

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Abstract: This work aims at clarifying the main theoretical aspects of the monopolistic competition Dixit-Stiglitz type; this is considered a reference point in economics theory, and also a basis in the prolific field, that is New Economics Geography. The subject helps to an extent the economic research in modeling the international trade. We must mention that in this paper we are focusing only on the demand.

Keywords: monopolistic competition; maximizing of utility; price index

General Framework

We consider that the economy consists of two territorial entities, which could be two regions, two countries, as well as two sectors, alleged industry and agricultural. The consumers from regions are represented by workers and farmers. The farmers received an income for the job done at the farms in the regions from where they are. They can play a double role: as farmers and as workers at the farms. As they are the owners are employees of that farm. The income flow of the farmers is a part of bilateral transfer: they are receiving an income from the owners, i.e. a wage; in turn they are offering a work, a service. Farmers' owners are using work of the farms, in order to produce agricultural goods under monopolistic competition conditions and increasing returns of scale. The agricultural goods are sold to the consumers from both regions and there are no transportation costs. Taking into account the presumption of perfect competition in this sector, there will be an agricultural good which is not differentiated. The work used in this sector is presumed to be immobile between regions. There is also an industrial sector, which consists of number of companies, each of them producing a different good, i.e. a single good for which they used only labor, under internal scale economies. These presumptions make the companies to act under monopolistic competition in order to establish the price of their products. Industrial goods are different, and each company activating in this sector are producing a single good, i.e. a single variety of industrial good. The trade between sectors assumes transport costs. The labor employed in this sector is mobile between sectors.

In order to view the assumptions is useful to draw the schematically diagram of the core-periphery model, where we insert only the types of competition as in figure 1.

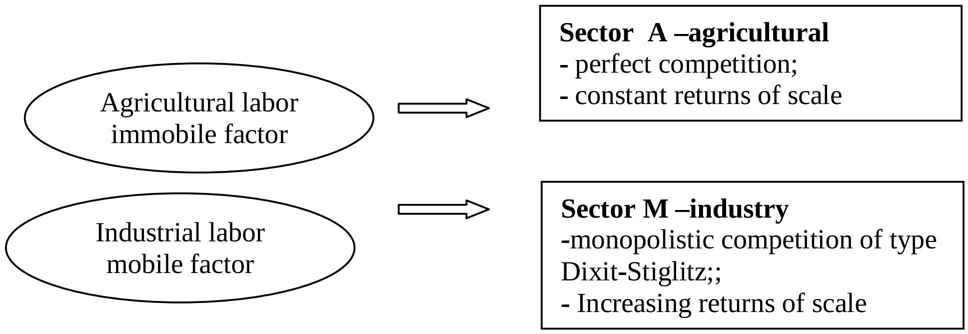

Figure 1. Diagram of the core-periphery model

The Demand

Consumers from both regions record preferences of type Cobb-Douglas for the two types of goods, as follows:

$$U = M^{\mu} A^{1-\mu};\ 0 < \mu < 1 \quad (1)$$

where:

M represents an aggregated by the form of an index of consume from industrial goods. A represents consumption from agricultural good, μ is the share from expenditure or from income with purchasing industrial good, and represents a fundamental parameter in core-periphery model. Implicitly, $(1-\mu)$ represents the share from consumers' income with purchasing agricultural good. It can be demonstrated the influence of the parameter μ, which represents an exogen variable in this model, by maximizing of the utility function of the consumers, U expressed by relation (1) under income restriction, which is supposed to be:

$$Y = G * M + p^A * A \quad (2)$$

Y represents income, G is a price index of industrial goods, $and\ p^A$ is the price of agricultural good. Under these circumstances it can be expressed a Lagrange function:

$$L = M^{\mu} A^{1-\mu} + \lambda\,(\,G * M + p^A * A - Y)\ (3)$$

which by partial derivation by M and A will give:

$$\frac{\partial L}{\partial M} = \mu M^{\mu-1} A^{1-\mu} + \lambda G = 0$$

$$\frac{\partial L}{\partial A} = (1-\mu) M^{\mu} A^{-\mu} + \lambda * p_A = 0\ (4)$$

Those relations cause:

$$M = \frac{\mu Y}{G};\ A = \frac{(1-\mu)Y}{p_A} \quad (5)$$

In other words, those relations demonstrate that it is optimum for consumers to allocate a share μ with purchasing industrial goods, and $(1-\mu)$ with agricultural goods.

Graphically, the amount of A and M are refined in place in what utility curves for various value of μ, are intersect with the line of income restriction; see the figures 2 and 3

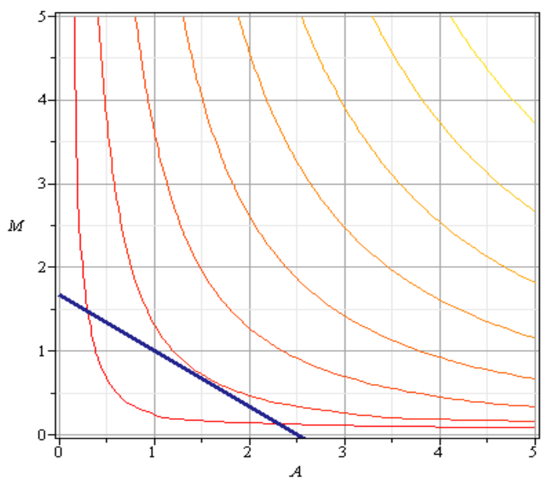

Figure 2. Utility for various amount of μ – share of expenditure with industrial goods

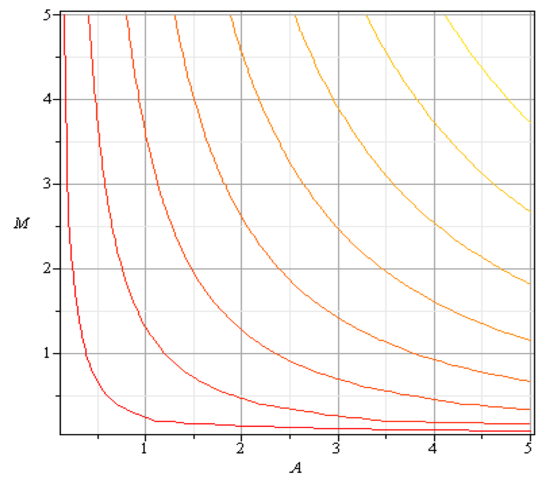

Figure 3. Utility for various amount of μ - share of industrial goods and income restriction

We are mentioning that all graphics and applications are done with Maple, version number 12.

Next we approach the way of choosing the industrial varieties, i.e. the manner in which the income allocated for industrial goods is spent, in order to purchase industrial varieties. The process is similar as the one above, the only difference is based on the fact that we shall assume that the quantity from industrial goods, M represents a subfunction over a continuity of variety from industrial goods. We shall note with m(i) consumption from each variety, generically noted with i, and the number of variety noted by n. Subfunction of M is a CES one, i.e. having a substitution coefficient between varieties constant. In such circumstances M can be defined as follows:

$$M = [\int_0^n m(i)^\rho di]^{\frac{1}{\rho}}\,; \quad M = (\sum_0^n m(i)^\rho)^{\frac{1}{\rho}};\; 0 < \rho < 1 \qquad (6)$$

The former definition is applied to continuous values of varieties, and the last form to discrete values; choosing of one or other, which will not affect our research in this paper.

ρ represents the intensity of preference for varieties from industrial good of the consumer. It is appreciated that as ρ goes to 1 industrial goods, differentiated are close substituible, and as for $\rho = 1$ are perfect substituible for any two varieties. Obviously, in the extreme case $M = \int_0^n m(i)di$ suggests the fact that variety does no matter for consumer utility. Otherwise, the consumer records the same level of utility, that is: either consums 50 units from a variety or one unit from 50 varieties. In this case, ρ goes to 0 the desire to consume a great variety from industrial goods increases. Location of ρ parameter between these cases, to be less, that means the varieties are imperfect substituible and to be positive meaning that the varieties are substituible and not complementary. By a notation we shall consider $\boldsymbol{\sigma}$ which is elasticity of substitution between any pair of varieties, which is constant and positive.

In order to establish the manner of allocation of the income between industrial varieties, we shall note by p(i) the price of one variety i, which is enclosed in field i=0..n, so as one consumer will have an income:

$$Y = p^A A + \int_0^n m(i)p(i)di \quad (7)$$

A consumer which has the target maximizing of consumption from industrial goods M, under the income restriction, or minimizing the cost in order to obtain industrial varieties, will register a function Lagrange as:

$$L(i) = \int_0^n m(i)p(i)di + \boldsymbol{\lambda}[[\int_0^n m(i)^\rho di]^{\frac{1}{\rho}} \text{-M}] \qquad (8)$$

for the good i, and respectively

$$L(j) = \int_0^n m(j)p(j)dj + \boldsymbol{\lambda}[[\int_0^n m(j)^\rho dj]^{\frac{1}{\rho}} \text{-M}] \qquad (9)$$

for the good j.

By partial derivation in function of m(i) and m(j) and equals with zero we will obtain:

$$p(i) = m(i)^{\rho-1}; \;\; p(j) = m(j)^{\rho-1} \;\; (10)$$

The relations (10) can be transformed in equality between marginal rate of substitution and prices of two varieties:

$$\frac{m(i)^{\rho-1}}{m(j)^{\rho-1}} = \frac{p(i)}{p(j)} \quad (11)$$

In the relation no. 6 if we express m(i) from the relation (11), and substituted in the initial restriction, (i.e. in relation no 6), then we will obtain:

$$m(j) = \frac{p(j)^{\frac{1}{(\rho-1)}}}{[\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{1}{\rho}}} M \;\; (12)$$

This relation (12) represents the function of compensated demand for variety j from industrial good. By transforming, we multiply both side with p(j), and integrating over all goods j we obtain:

$$m(j)p(j) = \frac{p(j)^{\frac{1}{(\rho-1)}+1}}{[\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{1}{\rho}}} M; \int_0^n m(j)p(j) = \frac{\int_0^n p(j)^{\frac{\rho}{(\rho-1)}} dj}{[\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{1}{\rho}}} M$$

$$\int_0^n m(j)p(j) = [\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{(\rho-1)}{\rho}} M \qquad (13)$$

It can be assess the right side of relation (13) as a product between quantity of industrial goods M and an aggregated index of the price. A variable G can be defined as:

$$G = [\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{(\rho-1)}{\rho}} = [\int_0^n p(i)^{(1-\sigma)} di]^{\frac{1}{(1-\sigma)}} \qquad (14)$$

In such circumstanes, the demand for a variety j can be expressed using relation (12):

$$m(j) = (\frac{p(j)}{G})^{\frac{1}{(\rho-1)}} M = (\frac{p(j)}{G})^{-\sigma} M \qquad (15)$$

But takin into account the relation (5), in which $M = \frac{\mu Y}{G}$,we obtain:

$$m(j) = (\frac{p(j)}{G})^{\frac{1}{(\rho-1)}}\frac{\mu Y}{G} = (\frac{p(j)}{G})^{-\sigma}\frac{\mu Y}{G} = \mu Y\frac{p(j)^{-\sigma}}{G^{-(\sigma-1)}} = \mu Y p(j)^{-\sigma}G^{(\sigma-1)}$$

for j$\in [0, n]$ (16)

As being written so, or under a particular form for a variety, let say 1, m(1)= $\mu Y p(1)^{-\sigma}G^{(\sigma-1)}$, the demand is related:

a) by the income spent for purchasing of the industrial goods μY;
b) by the price of variety j, p(j) or p(1);
c) by the amount of parameter $\boldsymbol{\sigma}$ elasticity of substitution between varieties;
d) by index price of industrial goods G.

The Ways of Action on Demand

The ways of action on demand are analyzed below:

a) Relation between demand of one industrial variety and income spend with purchasing of goods.

Studying relation (16) there can be noticed that m(j) –the demand of one individual good j is an homogeneous function of first degree with μY - the income allocated for the industrial goods, if the income increases with k % and the demand increases with same k %. This interaction could be seen in graphic no. 4, in which income increases two times, i.e. from 2 at 4 units, and the demand for the good j increases from 25 at 50 units.

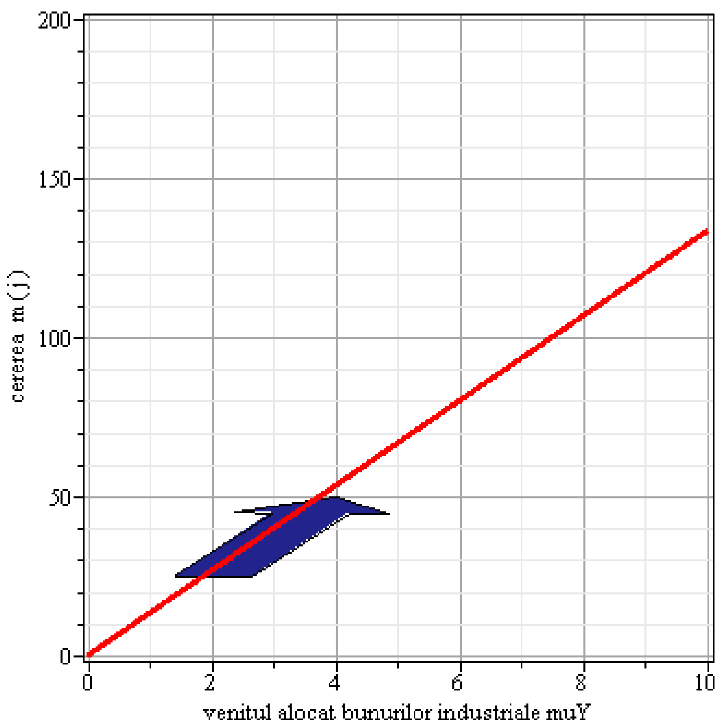

Figure 4. The Dependence between allocated income and demand

b) Dependence between price and demand of one variety

Relation (16) can be written so $m(j) = p(j)^{-\sigma}(G^{(\sigma-1)}\mu Y)$, which contains under paranthese an index price of all industrial goods G and the income allocated for purchasing industrial goods μ, both of them constantly assumed, because for an individual firm are macroeconomics indicators, so as the individual firm taken them as given. In such circumstances, the demand function for an individual variety j can be express as:

$$m(j) = const\ p(j)^{-\sigma}$$

and in particular for a variety 1 as:

$$m(1) = const\ p(j)^{-\sigma}$$

From relation (11) $\frac{m(i)^{\rho-1}}{m(j)^{\rho-1}} = \frac{p(i)}{p(j)}$, $\sigma = 1/(1-\rho)$ results $\left(\frac{\partial m(i)}{\partial m(j)} : \frac{m(i)}{m(j)}\right) : \left(\frac{\partial p(i)}{\partial p(j)} : \frac{p(i)}{p(j)}\right) = \boldsymbol{\sigma}$.

So, the elasticity of the demand depends on the fact that the price is a constant amount and equal with $\boldsymbol{\sigma}$. The dependence between price and demand for various amount of $\boldsymbol{\sigma}$, respectively for 2, 4 and 6 as in figure 5, in which it assumed constant equal with 150.

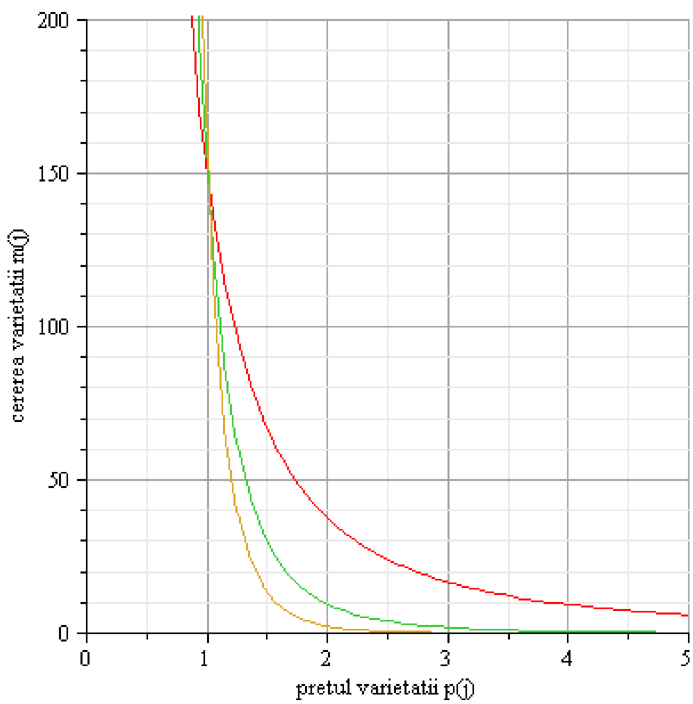

Figure 5. The dependence between demand and price for $\boldsymbol{\sigma}$ = 2,4 and 6; const=150

It may be observed that for a price equal with 1, the demand is the same regardless the elasticity of demand which depends on the price; also the demand for a certain variety reduces more and more as the price of the variety increases, if the elasticity is bigger.

c) Influence of $\boldsymbol{\sigma}$ parameter

Even this parameter was included in price, see point b), it is useful a detail because it measures many aspects in model. We are mentioning that the whole presentation of the monopolistic competition has on its base the contribution of Dixit-Stiglitz (1977).

Up to this moment it has resulted the fact that $\boldsymbol{\sigma}$ measures the elasticity of the demand, depending the price for an individual industrial variety, being a constant amount and also the elasticity of the substitution between the two varieties from industrial good. We are remembering that $\boldsymbol{\sigma}$ was defined in connection with ρ, the intensity for the variety of the consumers $\sigma = 1/(1-\rho)$ as $0 < \rho < 1$, which means:

$\boldsymbol{\sigma} > 1$.This dependence can be seen in following figure:

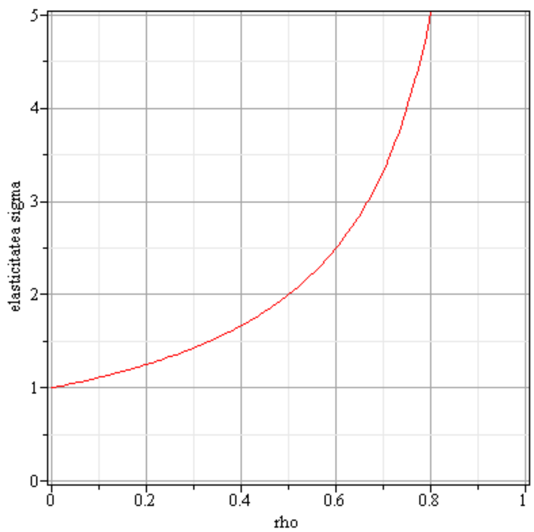

Figure 6. The dependence between parameters $\boldsymbol{\sigma}$ and ρ

d) Influence of the price index for industrial goods G

We have defined index price as: $G = [\int_0^n p(i)^{\frac{\rho}{(\rho-1)}} di]^{\frac{(\rho-1)}{\rho}} = [\int_0^n p(i)^{(1-\sigma)} di]^{\frac{1}{(1-\sigma)}}$, see relation (14) and also the demand for one variety m(j) is influenced by the index G, $m(j) = \mu Y p(j)^{-\sigma} G^{(\sigma-1)}$. The dependence between the price index G and the individual demand of one variety j, for different values of sigma, considering other of the factors unchanged, it can be seen in figure 7.

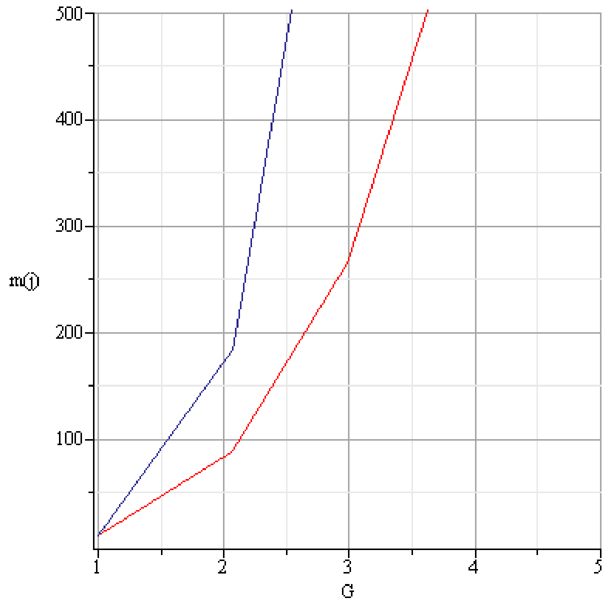

Figure 7. The dependence between price index G individual demand, for different amount of sigma, 4 and 5.

As it can be noticed there is a direct relation between index price G and the demand for one variety j m(j). The explanation is based on the fact that if index price G increases in average, then it records an increase of prices for varieties, which are competing with variety j, and so it will record an increase of the demand for the variety j, m(j). Otherwise, if the price of variety increases, then the demand decreases, but the demand for other varieties will increase, because the goods are substituible.

As it is defined the price index, if the industrial goods G depends by individual level of the price p(j), by the elasticity coefficient of substitution between varieties $\boldsymbol{\sigma}$ and by number of industrial varieties n. In order to view the way of actions, we assume that individual prices will be equal, i.e.

p(1)=p(2)=…………=p(n)=p(m).So, index price of industrial goods will be:

$$G = p(m)n^{\frac{1}{1-\sigma}} \qquad (17)$$

The price index of the industrial goods G depends directly on the individual price of industrial varieties, p(m).

The price index of the industrial goods G also depends on the elasticity of substitution between varieties $\boldsymbol{\sigma}$, and of the number of differentiated varieties n. So, as much as the elasticity of substitution $\boldsymbol{\sigma}$ reduces, or the more differentiated the varieties become, the bigger will be the reduction of the price index G, generated by an increase of number of varieties n. They can all be seen in figure 8.

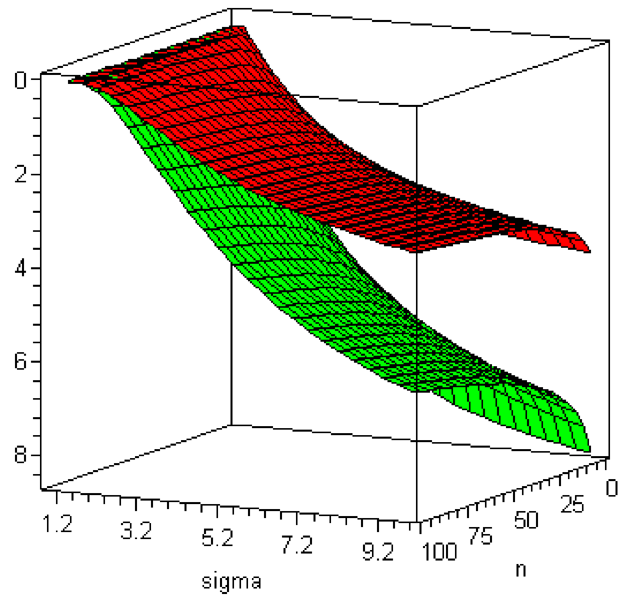

Figure 8. The dependence between sigma and n

In figure 8 there were drawn two levels of the individual prices, p(m)=5 (low curve), and 10 (up curve). According to the graphics, the price index of industrial goods G increases, as we move from up to down. That means G increases as the individual prices increase. Also, as much as the elasticity of substitution decreases $\boldsymbol{\sigma}$, in other words, as much the varieties become more differentiated, the greater will be the reduction recorded by the price index G, generated by an increase of varieties number n.

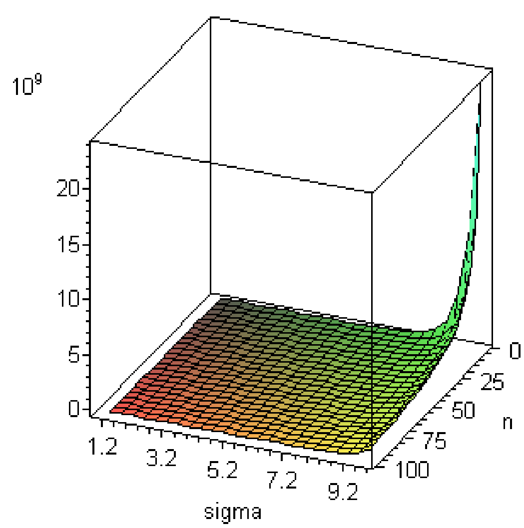

Figure 9. The dependence between sigma and n to individual demand m(j)

Conclusions

We consider that all theoretical aspects are useful for understanding the central model of New Economics Geography, which is the core-periphery duet of Paul Krugman. In the near future we shall extent our research towards the supply model.

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